

## Hayward's Primary School Calculation Policy 2022

## Hayward's Maths Vision

At Hayward's, we strive for all children to develop a positive attitude to Maths as an interesting and valuable subject; we do not want children to fear Maths yet have a belief that they can achieve highly.

We strive for all children to become confident when talking about their mathematics, using reasoning apparatus, diagrams and explanations - to show their understanding; we have high expectations of children's use of mathematical language.

We strive for all children to understand that Maths is not only the quick recall or computation of number facts - although these are fundamental - but that it is the considered thought and application using what they know to work through a problem; we want children to feel confident when solving problems.

We strive for all children to develop a range of efficient strategies - their own tool kits - and make choices about how to use their maths; we want children to make decisions and explain them.

We strive for all children to feel challenged but enthused by Maths; we want every child to enjoy and succeed as Mathematicians, understanding its very important place in our world.

## Aims

The national curriculum for mathematics aims to ensure that all pupils:

- become fluent in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately
- reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- can solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions


## Guidance for following the Calculation Policy

Written methods of calculations are based on mental strategies.
Each of the four operations builds on mental skills which provide the foundation for jottings and informal written methods of recording.
Skills need to be taught, practised and reviewed constantly. These skills lead on to more formal written methods of calculation.
Strategies for calculation need to be represented by models and images to support, develop and secure understanding. This, in turn, builds fluency.
When teaching a new strategy, it is important to start with numbers that the child can easily manipulate so that they can understand the methodology.
The transition between stages should not be hurried as not all children will be ready to move on to the next stage at the same time, therefore the progression in this document is outlined in stages.
Previous stages may need to be revisited to consolidate understanding when introducing a new strategy.

## A sound understanding of the number system is essential for children to carry out calculations efficiently and accurately.

## Our Methodology

At Hayward's, we want children to become confident in choosing the most efficient methods to solve a calculation. In many cases, children will have more than one method "up their sleeve" to find the solution to a calculation and will be able to pick the best one.
For children to be able to work with the columnar methods, we want children to have a sound understanding of place value. Many calculations should be solved mentally and/or with jottings; formal column methods are for calculations where the digits are varied and include regrouping or exchanging.

Correct Terminology

| Correct | Avoid |
| :---: | :---: |
| regrouping | carrying |
| exchanging | borrowing/stealing |
| ones | units |
| calculation / equation | sum |
| is equal to / the same as | equals |
| unknown | answer |

## Calculation policy: Addition and Subtraction

Key language for addition: sum, total, parts and wholes, plus, add, altogether, more, 'is the same as'
Key language for subtraction: take away, subtract, find the difference, fewer, less than

Addition and subtraction are inverse operations. Right from the start, children should be taught these as related operations. There are four number sentences (two using + and two using -) which can be written to express the relationship between 4 and 6 and 10. It is key to a good understanding of addition and subtraction that $6+[]=10$ and $10-6=[]$ are seen as ways of expressing the same question. Children should be encouraged to use inverse operations to check their solutions. Part-Whole models and bar models are visuals that show the relationship. It is also important to relate addition and subtraction to place value.

## A digit's true value should always be referenced.

Here is an example of language used to teach and talk through column addition:

- Approximately, what will $245+378$ be the same as? Let's use rounding to estimate.
- I have 5 ones and I am adding 8 ones so now I have 13 ones which I can regroup or repartition as 1 ten and 3 ones.
- Next, I have 6 tens and I am adding 7 tens but I also have 1 ten from regrouping which makes 14 tens. I can regroup this as 1 hundred and 4 tens.
- Finally: 2 hundreds add 3 hundreds add 1 hundred from regrouping makes 6 hundreds.
- $243+368$ is equal to 611 .
- How could I write this as a subtraction?


## Resources and Images for Addition and Subtraction

All learners should use resources and imagery to both develop and show understanding. Children should regularly draw pictures showing real life contexts for problems which they are solving. In order to help children remember certain strategies, certain references will be used throughout the school:


| 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: |
| 11 | 12 | 13 | 14 | 15 |
| 21 | 22 | 23 | 24 | 25 |
| 31 | 32 |  | 0 | 0 |
|  | 34 | 35 |  |  |
| 41 | 42 | 43 | 44 | 45 |

A spider may be used to help children understand the strategy of vertical jumps on a number square.


A frog may be used to help children understand the strategy of horizontal jumps on a numberline.

## EYFS

## Early Learning Goal

## Number

Have a deep understanding of number to 10 , including the composition of number
Subitise (recognised
Numerical Patterns

Apparatus is key in helping children to achieve the early learning goal. Children should be encouraged to show their understanding through the full range of apparatus. Children should also be encouraged to draw their own pictures to help to achieve and show their understanding.


## Year 1

## Addition

## Subtraction

## Using place value

Count on in ones/counting in tens, e.g. knowing $45+1$ or $45+10$ without counting on in ones.

$45+$ $\qquad$ $=50$
$65+$ $\qquad$ $=70$
$85+$ $\qquad$ $=90$

## Counting on

Count on in ones, e.g. $11+2=$ and $7+4=$
Count on in tens, e.g. $45+20$ as $45,55,65$

## Using number facts

'Story' of 4, 5, 6, 7, 8 and 9, e.g. $7=7+0$ or $6+1$ or $5+2$ or $4+3$
Number bonds to 10, e.g. $5+5,6+4,7+3,8+2,9+1,10+0$.

$6+\square=10$

| 10 |  |  |
| :--- | :--- | :---: |
|  | 5 |  |

$5+\square$
$=10$


Patterns using known facts, e.g. $4+3=7$ so we know $24+3,44+3$, $74+3$, etc.

Bead strings and 1-100 number grid help counting on/back in tens.

## Using place value

Count back in 1s/Count back in 10 s .
Say one less than any number to 100.
Say 10 less without counting back in ones.


| 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: |
| 11 | 12 | 13 | 14 | 15 |
| 21 | 22 | 23 | 24 | 25 |
| 31 | 32 | 9 | 9 | 34 |
| 41 | 42 | 43 | 44 | 45 |

$33-10=23$

Count back in ones,
e.g. $15-3=25-3=$


Count back in tens.


Using number facts
'Story' of 4, 5, 6, 7, 8 and 9, e.g. $7-1=6,7-2=5,7-3=4$, etc.
Number bonds to 10 , e.g. $10-1=0,10-2=8,10-3=7$, etc.

Missing number sentences,
$3+[]=7$, link addition
and subtraction


Patterns using known facts.

## Year 2

## Addition

## Subtraction

## Using place value

Know 1 more or 10 more than any number, e.g. 1 more than 67 or 10 more than 85.
Partitioning, e.g. $55+37$ as $50+30$ and $5+7$ finally combining the two totals: 80 + 12 .


Bead strings and 1-100 number grid help counting on/back in tens.

## Using place value

Know 1 less or 10 less than any number, e.g. 1 less than 74 or 10 less than 82.
Partitioning. e.g. 55-32 as 50-30
and 5-2 combining the answers: $20+3$.


## Counting on

Add ten and mupltiples of ten. e.g. $76+20$ as 76.86 .96 or in one hop $76+20$. Add two 2 -digit numbers by counting on in tens and then in ones. e.g. $55+37$ as 55 add 30 (85) add 7 (92).


Add near multiples. e.g. $46+19$ or $63+21$.

## Using number facts

Know pairs of numbers which make the numbers up to and including 10 . e.g. $8=4 \& 4,3 \& 5,2 \& 6,1 \& 7$ and $10=5 \& 5,4 \& 6,3 \& 7,2 \& 8,1 \& 9,0 \& 10$. Patterns of known facts, e.g. $6+3=9$, so we know $36+3=39$. $66+3=69.53+6=59$.
Bridging ten, e.g. $57+5$ as 57 add 3 then add 2 more.


> Missing number sentences.
> $3+[]=7$. link addition and subtraction.

## Taking away

Subtract ten and multiples of ten, e.g. 76-20 as 76. 66, 56 or in one hop $76-20=56$. Subtract two 2 -digit numbers by counting back in tens then in ones, e.g. 67-33 as 67 subtract 30 (37) then count back 3 (34).


Subtracting near multiples. e.g. 74-21 or 57-19.

## U:ing number fact:

Know pairs of numbers which make the numbers up to and including 10.
e.g. $10-6=4.8-3=5.5-2=3$. etc.

Patterns of known facts. e.g. $9-6=3$, so we know 39-6=33. 69-6 $=63$. $89-6=83$.

Bridge ten, e.g. 52-6 as 52 subtract 2 then subtract 4 more.


Adding three or more single-digit numbers, spotting bonds to 10 or doubles. e.g. $6+7+4+2$ as $10+7+2$.

## Counting up

Find a difference between two numbers on a line. e.g. 51-47.

## Addition

## Subtraction

## Using place value

Count in hundreds, e.g. knowing $475+200$ as $\mathbf{4 7 5}, 575,675$.
Add multiples of 10,100 and $£ 1$
e.g. $746+200$ or $746+40$ or
$£ 6.34+£ 5$ as $£ 6+£ 5$ and 34 p.
Partitioning, e.g. $68+74$ as $60+70$ and $8+4$ and combine the totals: $130+12=142$
Or $£ 8.50+£ 3.70$ as $£ 8+£ 3$ and $50 p+70$ p and combine: $£ 11+£ 1.20$.

## Counting on

Add 2-digit numbers by adding the multiple of ten then the ones,
e.g. $67+55$ as 67 add 50 (117) add 5 (122).

Add near multiples of 10 and 100 , e.g. $67+39$ or $364+199$.


Count on from 3-digit nos, e.g. $247+34$ as $247+30$ (277)
then $277+4=281$.

## Using number facts

Number bonds to 100 , e.g. $35+65,46+54,73+27$, etc.

| 100 |  |
| :---: | :---: |
| 65 | 35 |

We no longer count in ls but use PV and number facts.

## Taking away

Use place value to subtract, e.g. 358-300 or 348-40 or 348-8.
Taking away multiples of 10,100 and $£ 1$, e.g. $476-40=436$.

$$
476-300=176, £ 4.76-£ 2=£ 2.76
$$

Partitioning, e.g. 68-42 as $60-40$ and $8-2$ or $£ 6.84-£ 2.40$ as $£ 6-£ 2$ and $80 p-40$ p.
Count back in hundreds, tens and then ones.
e.g. $763-121$ as $763-100(663)$ then subtract $20(643)$ then subtract 1 (642).
Subtract near multiples, e.g. 648-199 or 86-39.
86-39


## Counting up

Find a difference between two numbers by counting up from the smaller to the larger, e.g. 121-87.


## Using number facts

Number bonds to 100 , e.g. $100-35=65,100-48=52$, etc.

| 100 |  |
| :---: | :---: |
| 48 | $?$ |

Add to next ten and next hundred, e.g. $176+4=180,435+65=500$, etc.

## Year 3

## Written Addition

## Written Subtraction

## Written methods

 with two 3-digit numbers.

Expanded column addition with 'carrying'.

| 400 | 60 | 6 |
| ---: | :--- | :--- |
| +300 | 50 | 8 |
| 800 | 20 | 4 |
| 100 | 10 |  |

Compact column addition with two or more 3 -digit numbers or towers of 2 -digit numbers.

$$
+\begin{array}{r}
47 \\
76 \\
123
\end{array}
$$

Compact column addition with 3 -digit numbers
Recognise fractions which add to 1. e.g. $\frac{1}{4}+\frac{3}{4}=1$ or $\frac{2}{5}+\frac{3}{5}=1$

Build on partitioning to develop expanded column addition
Develop counting up subtraction.

| 400 | 60 | 6 |
| ---: | ---: | :---: |
| +300 | 50 | 8 |
| 700 | 110 | 14 |

3

Use counting up subtraction to find change from $£ 1$ and $£ 10$.


Expanded column subtraction, moving onto exchanging when ready: 258-73 = 185
Important to see the visual image of fractions totalling one whole.

$$
\begin{array}{r}
200+50+8 \\
-\quad 70+3 \\
\hline
\end{array}
$$

becomes $\frac{-\quad 70+3}{100+80+5}$

## Addition

## Subtraction

## Using place value

Count in thousands, e.g. knowing $475+200$ as $475,575,675$.
Partitioning, e.g. $746+203$ as $700+200$ and $46+3$
or $134+707$ as $130+700$ and $4+7$.

## Counting on

Add 2-digit numbers by adding the multiple of ten then
the ones, e.g. $67+55$ as 67 add 50 (117) add 5 (122).
Add near multiples of 10,100 and 1000 e.g. $467+199$ or $3462+2999$.


Count on to add 3-digit numbers and money, e.g. $463+124$ as $463+100(563)+20(583)+4=587$ or $£ 4.67+£ 5.30$ as $£ 9.67$ add 30 p.

## Using number facts

Number bonds to 100 and to next multiple of 100 .
e.g. $463+37,1353+47$.

Number bonds to $£ 1$ and to the next whole pound, e.g. $£ 3.45+55$ p.

Add to the next whole number, e.g. $4.6+0.4$ or $7.2+0.8$.

PV and number facts are central to mental strategies.

## Taklng away

Use place value to subtract, e.g. 4748-4000 or 4748-8, etc.
Take away multiples of $10,100,1000, £ 1,10$ p or 0.1 , e.g. $8392-50$ or 6723-3000 or £3.74-30p or 5.6-0.2.

Partioning, e.g. £5.87-£3.04 as $£ 5-£ 3$ and $7 p-4 p$ or 7493-2020 as 7000-2000 and 90-20.

Count back, e.g. 6482-1301 as 6482-1000, then - 300, then -1 (5181) Subtract near multiples, e.g. 3522-1999 or $£ 34.86-£ 19.99$.

## Counting up

Find a difference between two numbers by counting up from the smaller to the larger, e.g. 506-387.

$100+10+6+3-119$

## Using number facts

Number bonds to 10,100 and derived facts.
e.g. $100-76=24,1.0-0.6=0.4$.

> Qounting up is essential for money calculations and. later, decimals.

| 100 |  |
| :---: | :---: |
| 76 | 24 |

Number bonds to $£ 1$ and $£ 10$.
e.g. $£ 1.00-86 p=14$ por $£ 10-£ 3.40=£ 6.60$.

## Year 4

## Written Addition

## Written Subtraction

Build on expanded column addition to develop compact column addition with larger numbers.

| 1000 | 400 | 60 | 8 |
| ---: | :---: | :---: | :---: |
| + | 4000 | 800 | 60 |
| 6000 | 300 | 30 | 4 |
| 1000 | 100 | 10 |  |

Compact column addition with larger numbers.

$$
\begin{array}{r}
5347 \\
2286 \\
+1495 \\
\hline 9128 \\
\hline 121
\end{array}
$$

Use expanded and compact column addition to add amounts of money. e.g. $£ 3.24+£ 2.58$.


Add like fractions, e.g. $\frac{3}{8}+\frac{2}{8}+\frac{1}{8}$

Expanded column subtraction.

| 600 | 110 | 16 |
| ---: | ---: | ---: |
| 700 | 20 | 6 |
| -300 | 50 | 8 |
| 300 | 60 | 8 |

Begin to use column subtraction.


Use counting up subtraction to find change from $£ 10, £ 20, £ 50$ and $£ 100$. e.g. $£ 100-£ 73.60$.

$£ 20+£ 6+40 p=£ 26.40$

Subtract like fractions, e.g $\frac{7}{8}=\frac{2}{8}+\frac{5}{8}$

## Year 5

## Addition

## Using place value

Count in $0.1 \mathrm{~s}, 0.01$ s, e.g. knowing what 0.1 more than 0.51 is.
Partitioning, e.g. $2.4+5.8$ as $2+5$ and
$0.4+0.8$ and combine the totals: $7+1.2=8.2$.


## Counting on

Add two decimal numbers by adding the ones then the tenths/hundredths. e.g. $5.72+3.05$ as 5.72 add $3(8.72)$ then add 0.05 (8.77).

Add near multiples of 1 , e.g. $6.34+0.99$ or $5.63+0.9$.


Count on from large numbers, e.g. $6834+3005$ as $9834+5$.

## Using number facts

Number bonds to 1 and to the next whole number, e.g. $0.4+0.6$ or $5.7+0.3$.

Add to the next ten from a decimal number. e.g. $7.8+2.2=10$.


Subtracting by counting up is much less error prone.

## Subtraction

## Taking away

Using place value to subtract decimals, e.g. 4.58-0.08 or 6.26-0.2, etc. Take away multiples of powers of 10, e.g. 15, 672-300 or 4.82-2 or 2.71-0.5 or 4.68-0.02.

Partition or count back, e.g. 3964-1051 or 5.72-2.01.
Subtract near multiples, e.g. 86.456-9999 or 3.58-1.99.

## Counting up

Find a difference between two numbers by counting up from the smaller to the larger, e.g. 2009-869.

ind change using shopkeepers' addition, e.g. buy toy for $£ 6.89$ using $£ 10$.


## Using number facts

Derived facts from number bonds to 10 and 100, e.g. $2-0.45$ using $45+55=$ or 100 or $3.00-0.86$ using $86+14=100$.

Knowledge of number bonds underpins mental stratgegies.

| 100 |  |
| :--- | :--- |
| 86 | 14 |

Number bonds to $£ 1 . £ 10$ and $£ 100$, e.g. $£ 4.00-£ 3.86 p=14$ p or $£ 100-£ 66$ using $66+34=£ 100$.

## Year 5

## Written Addition

## Written Subtraction

Expanded column addition for money leading to compact column addition for adding several amounts of money.

| $£ 14$ | $60 p$ | $4 p$ |  |
| ---: | :--- | :--- | :--- |
| $£ 28$ | $70 p$ | $8 p$ |  |
| $+£ 12$ | $20 p$ | $6 p$ |  |
| $£ 55$ | $60 p$ | $8 p$ | $£ 55.68$ |

Compact column addition to add pairs of 5-digit numbers.
Continue to use column addition to add towers of several larger numbers.
Use compact addition to add decimal numbers with up to two places.

Adding fractions with related denominators,

$$
\frac{3}{4}+\frac{1}{8}=\frac{7}{8}
$$

Expanded version first embeds understanding of place value.

Compact column subtraction for numbers with up to 5 digits. e.g. 16.324-8516.

$$
\begin{array}{rrrrr}
0 & 15 & 13 & 1 & 14 \\
-\chi & 6 & z & 2 & 4 \\
& 8 & 5 & 1 & 6 \\
\hline 7 & 8 & 0 & 8 \\
\hline
\end{array}
$$

Continue to use counting up subtraction for subtractions involving money. including finding change or, e.g. $£ 50-£ 28.76$.


Use counting up subtraction to subtract decimal numbers. e.g. 4.2-1.74.

$$
2+0.26+0.2=\underline{2.46}
$$

Equivalent fractions are the basis for + and fractions.

Subtracting fractions with related denominators,

$$
\text { e.g. } 1 \frac{1}{4}-\frac{3}{8}=1 \frac{2}{8}-\frac{3}{8}=\frac{10}{8}-\frac{3}{8}=\frac{7}{8}
$$

Number lines are useful when working with time. You don't have to think about the columns.


## Year 6

## Addition

## Subtraction

## Using place value

Count in 0.1 s, 0.01 s, 0.001 s, e.g. knowing what 0.001 more than 6.725 is. Partitioning, e.g. $9.54+3.25$ as $9+3$ and
$0.5+0.2$ and $0.04+0.05$ to get 12.79 .

| 10 s | 1 s | $\cdot$ | $0.151 / 10 \mathrm{~s}$ | $0.0151 / 100 \mathrm{~s}$ |
| :---: | :---: | :---: | :---: | :---: |
|  | 9 | $\cdot$ | 5 | 4 |
|  | 3 | $\cdot$ | 2 | 5 |
| 1 | 2 | $\cdot$ | 7 | 9 |

## Counting on

Add two decimal numbers by adding the ones then the tenths/hundredths or thousandths, e.g. $6.314+3.006$ as 6.314 add $3(9.314)$ then add 0.006 (9.32).

Add near multiples of 1 , e.g. $6.345+0.999$ or $5.673+0.9$.


Count on from large numbers, e.g. $16,375+12,003$.

## Using number facts

Number bonds to 1 and to the next multiple of 1 .
e.g. $0.63+0.37$ or $2.355+0.645$.

Add to next ten, e.g. $4.62+0.38$.

## Taking away

Use place value to subtract decimals, e.g. 7.782-0.08 or $16.263-0.2$, etc.
Take away multiples of powers of 10 , e.g. 132,956-400 or
686,109-40,000 or 7.823-0.5.
Parition or count back, e.g. 3964-1051 or 5.72-2.01.
Subtract near multiples, e.g. 360.078-99.998 or 12.831-0.99.

## Counting up

Count up to subtract numbers from multiples of 10, 100, 1000, 10,000 Find a difference between two decimal numbers by counting up from the smaller to the larger, e.g. 1.2-0.87.


## Using number facts

Derived facts from number bonds to 10 and 100, e.g. $0.1-0.075$
using $75+25=100$ or $5-0.65$ using $65+35=100$.
Number bonds to $£ 1, £ 10$ and $£ 100$. e.g. $£ 7.00-£ 4.37$ or $£ 100-£ 66.20$ using $20 \mathrm{p}+80 \mathrm{p}=£ 1$ and $£ 67+£ 33=£ 100$.

| $£ 100$ |  |
| :--- | ---: |
| $£ 67$ | $£ 33$ |

## Year 6

## Written Addition

## Written Subtraction

Compact column addition for adding several large numbers and decimals with up to two places.

Compact column addition with money.


Compact column subtraction for large numbers.


Adding fractions with unlike denominators

$$
\begin{array}{lc}
\frac{3}{4}+\frac{1}{3}=1 \frac{1}{12} \text { or } 2 \frac{1}{4}+1 \frac{1}{3}=3 \frac{7}{12} \\
\frac{3}{4}+\frac{1}{3} & \frac{9}{4}+\frac{4}{3}
\end{array}
$$

$$
=\frac{9}{12}+\frac{4}{12}
$$

$$
=\frac{27}{12}+\frac{16}{12}
$$

$$
=\frac{13}{12}
$$

$$
=\frac{43}{12}
$$

$=1 \frac{1}{12}$

Use counting up subtraction when dealing with money. e.g. $£ 100-£ 78.56$ or $£ 45.23-£ 27.57$.


Use counting up subtraction to subtract decimal numbers. e.g. 13.1-2.37.
fractions is
here.

$=\frac{5}{4}-\frac{2}{3}$
Subtracting fractions with unlike denominators,
$=\frac{15}{12}-\frac{8}{12}$

$$
=3 \frac{7}{12}
$$

$=\frac{7}{12}$

## Calculation policy: Multiplication and Division

Key language for multiplication: multiply, times, factor, multiple, product, groups of, lots of, equal Key language for division: divide, share, group, sharing, grouping, equal

Multiplication and division are inverse operations. Right from the start, children should be taught these as related operations. There are four number sentences (two using $x$ and two using $\div$ ) which can be written to express the relationship between 5 and 9 and 45 . It is key to a good understanding of division that [] $\times 5=45$ and $45 \div 5=[]$ are seen as ways of expressing the same question. Like in addition and subtraction, equations can be written with the "equals" symbol not necessarily at the end of an equation: $5=45 \div 9$.
The product is the number made when two (or more) numbers are multiplied together. Children should use the word product from Key Stage One. Factors are the numbers that can be multiplied to make a product. It is also important to relate multiplication and division to place value.

## A digit's true value should always be referenced.

Here are some examples of how linking back to place value can support the teaching of multiplication and division.
$40 \times 6=$
4 ones $\times 6=24$ ones so that means that 4 tens $\times 6=24$ tens which is 240 .
$3600 \div 9=$
3600 is the same as 36 hundreds. 36 hundreds $\div 9=4$ hundreds

## Progression

## Step

1 Equal groups (Size)
2 How many equal groups (Number)
3 Combining size and number of groups
4 Inefficiency of repeated addition $\longrightarrow$ Multiplication
5 Notation and language of multiplication
6 Multiplication is Commutative
7 Securing times tables facts
8 Use variation to explore and deepen the concept (inc generalisation)
9 Written multiplication

## Factors, Multiples, Primes, Squares and Cubes

Throughout primary school, children encounter special numbers. Children should know and use the vocabulary and be confident identifying and working with the numbers relevant to their year group.

A multiple is a number that can be divided by another number without a remainder. A multiple of 3 is a number in the $3 x$ table, a number in the pattern of counting in 3 s .

A factor is a number which can be multiplied with another to produce another number.
Factors come in pairs, unless the number is a square number.
Multiples and factors are best explained by using a number sentence such as the following:


A prime number is a number which has only 2 factors, 1 and itself.
It does not appear in any multiplication tables other than its own.
A square number is a number produced when a number is multiplied by itself.
$3 \times 3=9$ so 9 is a square number. Square numbers have an odd number of factors for this reason.
A cubed number is a number produced when a number is multiplied by itself and itself again.
$3 \times 3 \times 3=27$ so 27 is a cubed number.

## Resources and Images for Multiplication and Division

All learners should use resources and imagery to both develop and show understanding. Particularly in Key Stage One, where there are no formal methods for multiplication and division, children will be working with concrete objects and pictures. Children will record their work and show their methods by drawing pictures.
An important first step in the understanding of multiplication and division is understanding the concept of equal groups. Children will first use concrete objects and pictures to show their understanding of this.



Children will use objects and pictures to show their workings, especially in Key Stage One.


For "clever counting" (counting in steps) a frog may be used to help children understand the strategy of horizontal jumps on a numberline.

## Year 1

## Multiplication

## Counting In steps ('Clever' countingl

Count in 2 s and 10 s .


## Doubling and halving

Find doubles to double 6
using fingers.


## Grouplng

Begin to use visual and concrete arrays and 'sets of' objects

## Doubling and halving

Find half of even numbers up to 12 including realising that it is hard to halve an odd number.

## Couting In steps ('Clever' countingl <br> Count in 2 s , and 10 s .



## Division


 There are 5 balloons in a group. There are 4 groups.
$5+5+5+5=20$
4 lots of $5=20$


2 counters are in a row.
There are 4 rows.
How many counters are there? $2+2+2+2=8$

Children should be unitising to find totals: counting in groups, working towards using known facts. Addition: $(3)+(3)+(3)+(3)+(3)=(15)$ blocks Multiplication: ( $\mathbf{3}) \times(5)=(15$ )blocks

Some children may begin to use the multiplication symbol.

## Grouping

Begin to use visual and concrete arrays and 'sets of' objects to find the answers to 'how many towers of 3 can I make with 12 cubes?'

Sharing
Begin to find half of a quantity using sharing, e.g half of 16 cubes by giving one each repeatedly to two children.

## Division must be presented as the inverse of multiplication (grouping).

-2

Some children | 8 |  |  |  |
| :---: | :---: | :---: | :---: |
|  |  |  |  | may begin to use the division symbol.

## Year 2

## Multiplication

## Division

## Counting in steps 'Clever' countingl

Count in 2 s .5 s and 10 s .


Begin to count in 3s.
Doubling and halving
Begin to know doubles of multiples of 5 to 100.
eg. double 35 is 70
Groupling
Use stem sentences:
There are $\qquad$ pencils in each packet. There ars $\qquad$ packets.
$10+10+10=10 \times 3=30$


Use arrays to explore the commutative nature of multiplication. $5+5+5=5 \times 3=3 \times 5$.
3 lots/groups of 5 is equal to 5 lots/groups of 3 .

Use number facts
Know doubles to double 20

$$
\text { Double } 7=14
$$



Start learning $2 x, 5 x, 10 x$ tables, relating these to 'Clever counting in $2 x .5$ s, and 10 s. eg $5 \times 10=50$, and $10,20,30,40,50$ is five steps in the tens count.

## Counting in steps 'c'Clever' countingl

Count in 2 s. 5 s and 10 s
'Clever' counting is an excellent basis for multiplication and division.


## Doublling and having

Find half of numbers up to 40 , including realsing that half of an odd number gives a remainder of 1 or an answer containing a $1 / 2$
Begin to know half of multiples of 10 to 100 . eg half of 70 is 35 .

## Groupling

Relate division to multiplication by using arrays, objects and pictures. e.g I have 20 sausages and I put 4 on each plate, many plates do $I$ need? How many 4 s are in 20?

## Sharing

Relate division to the sharing of a total into a given number of equal groups. Explore with concrete objects, materials, pictures and arrays.
e.g the teacher has 30 pencils to share between 3 children. How many pencils do they get each?
The children will share the pencils equally between 3 groups.

## Using number facts

Know halves of even numbers to 24 . Know $2 x .5 \mathrm{x}$ and 10 x division facts.
half of 20 is...

Begin to know $3 x$ division facts.

| 20 |  |
| :---: | :---: |
| $?$ | $?$ |

## Mental Multiplication

## Mental Division

## Counting In steps ('Clever' countingl

Count in 2 s s $3 \mathrm{~s}, 4 \mathrm{~s}, 5 \mathrm{~s}, 8 \mathrm{~s}$ and 10 s , e.g. colour the multiples on a 1-100 grid or use hops along a landmarked line.


Doubling and halving
Find doubles to double 50 using partitioning.
Use doubling as a strategy in multiplying by 2 .
e.g. $18 \times 2$ is double 18 (36).


$$
\begin{aligned}
& \text { Doubling and halving form the } \\
& \text { basis of mental } \mathrm{x}: \div \text { strategies. }
\end{aligned}
$$

Doubling and halving

Counting in steps ('Clever' countingl
Count in $2 \mathrm{~s}, 3 \mathrm{~s}, 4 \mathrm{~s}, 5 \mathrm{~s}, 9 \mathrm{~s}$ and 10 s by colouring numbers on the 1-100 grid or using a landmarked line.


Find half of even numbers to 100 using partitioning.
Use halving as a strategy in dividing by 2 .
e.g. $36 \div 2$ is half of 36 .


Grouplng
Recognise that multiplication is commutative, e.g. $4 \times 8=8 \times 4$.
Multiply multiples of 10 by single-digit numbers, e.g. $30 \times 8=240$.
Multiply friendly 2 -digit numbers by single-digit numbers, e.g. $13 \times 4$.


## Grouplng

Recognise that division is not commutative, e.g. $16 \div 8$ does not equal $8 \div 16$. Relate division to multiplications' with holes in', e.g. $\square \times 5=30$ is the same calculation as $30 \div 5=$ ? thus we can count in in 5 s to find the answer. Divide multiples of 10 by single-digit numbers, e.g. $240 \div 8=30$.

Using number facts
Know halves of even numbers to 40 .

| 28 |  |
| :---: | :---: |
| $?$ | $?$ |

Know halves of multiples of 10 to 200 , e.g. half of 170 is 85 .
Know $2 \mathrm{x}, 3 \mathrm{x}, 4 \mathrm{x}, 5 \mathrm{x}, 8 \mathrm{x}, 10 \mathrm{x}$ division facts.
Use divison facts to find unit and simple non-unit fractions of amounts within the times tables, e.g. $1 / 4$ of 48 is $3 \times(48 \div 4)$.

## Written Multiplication

## Written Division

Using arrays to build understanding and make connections Use arrays to help children understand the relationships between calculations.
$2 \times 3=64 \times 3=12$
Encourage the exploration of arrays to make connections - that $4 \times 3$ is double $2 \times 3$ so the product is also double.
What would $8 \times 3$ be? Why?
What would $16 \times 3$ be? Why?

Progression towards grid multiplication
Use arrays to help children understand what the grid method is.


Build on partitioning to develop grid multiplication.

Lead towards partitioning into the tens and the ones as in the abstract grid method.
Children can explore partitioning the number in different ways, helping with their understanding of how multiplication tables link together.

| x | 20 | 3 | $=$ |
| :---: | :---: | :---: | :---: |
| 4 | 80 | 12 | 92 |

## Connect division to multiplication to solve division calculations

$84 \div 3=$ How many $3 s$ in 84 ?
Use known facts, with numberline to reinforce understanding.
$3 \times 10=30$
$3 \times 10=30$
$3 \times 8=24$
$3 \times 28=84$ so $84 \div 3=28$


Children can explore with different numberlines to find different ways of solving calculations.

$$
3 \times 30=90 \text { subtract } 3 \times 2=6=84
$$

Connect fractions to division
Understand that fractions relate to division - that tenths is dividing by 10 , quarters is dividing by 4 etc
Children will use division methods to find fractions of small amounts:
$\frac{4}{10}$ of $60=60 \div 10 \times 4$


## Mental Multiplication

## Counting in steps (sequences)

Count in $2 \mathrm{~s}, 3 \mathrm{~s}, 4 \mathrm{~s}, 5 \mathrm{~s}, 6 \mathrm{~s}, 7 \mathrm{~s}, 8 \mathrm{~s}, 9 \mathrm{~s}, 10 \mathrm{~s}, 11 \mathrm{~s}, 12 \mathrm{~s}, 25 \mathrm{~s}, 50 \mathrm{~s}, 100 \mathrm{~s}$ and 1000 s .

## Doubling and halving

Find doubles to double 100 and beyond using partitioning,
e.g. double 226.


Begin to double amounts of money,
e.g. $£ 3.50$ doubled is $£ 7$.

Use doubling as a strategy in multiplying by 2,4 and 8,
e.g. $34 \times 4=$ double 34 (68) doubled again (136).

## Grouplng

Use partitioning to multiply 2-digit numbers by single-digit numbers.
Multiply multiples of 100 by single-digit numbers using tables facts, e.g. $400 \times 8=3200$.

Use dienes to show the relationship with place value.
4 ones $\times 8=32$ ones so $400 \times 8=3200$ because:
4 hundreds $\times 8=32$ hundreds which is 3200 ones, so 3200 .
Use the abstract grid method to show multiplying by rounding
e.g. $7 \times 19=7 \times 20-7$

$$
\begin{aligned}
& \text { Facility in doubling and } \\
& \text { halving is key for mental } \\
& x \text { and } \div \text { strategies. }
\end{aligned}
$$

## Mental Division

## Counting in steps (sequences)

Count in $2 \mathrm{~s}, 3 \mathrm{~s}, 4 \mathrm{~s}, 5 \mathrm{~s}, 6 \mathrm{~s}, 7 \mathrm{~s}, 8 \mathrm{~s}, 9 \mathrm{~s}, 10 \mathrm{~s}, 11 \mathrm{~s}, 12 \mathrm{~s}, 25 \mathrm{~s}, 50 \mathrm{~s}, 100 \mathrm{~s}$ and 1000 s .

## Doubling and halving

Find halves of even numbers to 200 and beyond using partitioning.

| 344 |  |
| :---: | :---: |
| 172 | 172 |

Begin to half amounts of money, e.g. $£ 9$ halved is $£ 4.50$.
Use halving as a strategy in dividing by 2,4 and 8 , e.g. $164 \div 4$ is half of 164 (82) halved again (41).

## Groupling

Use multiples of 10 times the divisor to divide by numbers $<9$ above the tables facts, e.g. $45 \div 3$.


Divide multiples of 100 by single-digit numbers using division facts. e.g. $3200 \div 8=4000$.

## Using number facts

Know times tables up to $12 \times 12$ and all related division facts.
Use division facts to find unit and non-unit fractions of amounts within the times tables, e.g. $7 / 3$ of 56 is $7 \times(56 \div 8)$.

## Written Multiplication

## Written Division

Use grid multiplication to multipy 3-digit by 1-digit numbers.

| x | 200 | 50 | 3 |
| :---: | :---: | :---: | :---: |
| 6 | 1200 | 300 | 18 |$=1518$

## Formal Short Multiplication

Move from grid method to short multiplication to multiply 3-digit numbers by 1 -digit numbers.

## $127 \times 6=762$

127

| $\times \quad 6$ |
| :--- |

42 (6x7)
120 (6x20)
600 (6x100) 762

127
$\times 6$
$\frac{762}{14}$

Use the expanded column method as a bridge from the grid method to the formal short method to demonstrate to children the place value. Use it as a teaching point towards children learning the short method.

Use the language of place value to ensure understanding.
$6 \times 7$ ones is 42 ones, which is 4 tens and 2 ones, so the 4 is placed in the tens column and the 2 in the ones column.
$6 \times 2$ tens is 12 tens, but we also have 4 tens already, making 16 tens. 16 tens is 1 hundred and 6 tens, so we place the 1 in the hundreds column.
$6 \times 1$ hundred is 6 hundreds, plus the extra hundred we already have so we have 7 hundreds.

Ensure the digits that are regrouped are written under the line in the correct column, smaller than the actual digits in the calculation.

Connect division to multiplication to solve division calculations
Build on work from year 3 to continue to understand division as the inverse of multiplication, using known facts to solve calculations.

If children understand place value they can develop fluency.
$252 \div 3=$ How many 3 s in 252 ?
Use known facts and knowledge of partitioning, with a numberline to reinforce understanding.


Introduce the formal division layout
Using numbers under 100, introduce children to formal division, at first using partitioning and known facts.

$$
7 \longdiv { 7 0 + 2 8 } \quad \mathbf { 9 8 } \div \mathbf { 7 = 1 4 } \quad 7 \longdiv { 1 4 }
$$

Use a place value grid for dividing 1 digit numbers by 10 and 100
Identify the value of the digits in each number as ones, tenths and hundredths.


## Year 5

## Mental Multiplication

## Doubling and halving

Double amounts of money using partitioning. e.g. $£ 6.73$ doubled is double $£ 6$ ( $£ 12$ ) plus double 73 p ( $£ 1.46$ ).
Use doubling and halving as a strategy in multiplying by 2, 4, 8, 5 and 20 ,
e.g. $58 \times 5=1 / 2$ of 58 (29) $\times 10$ (290).


Grouplng
Multiply decimals by 10, 100, 1000, e.g. $3.4 \times 100=340$.


Use partitioning to multiply friendly 2 -digit and 3 -digit numbers by single-digit numbers, e.g. $402 \times 6$ as $400 \times 6$ (2400) and $2 \times 6$ (12).
Develop understanding of commutative law, associative law and distributive law to find "easy" routes to solve calculations
e.g. $5 \times 42 \times 4=42 \times 20=42 \times 10 \times 2$
$25 \times 84=84 \times 100 \div 4=8400 \div 4$

## Using number facts

Use times tables facts up to $12 \times 12$ to multiply multiples of the multiplier,
$6 \times 4=24$ so $0.6 \times 4=6 \times 4 \div 10=2.4$

## Mental Division

## Doubling and halving

Halve amounts of money using partitioning, e.g. half of $£ 14.84$ as half of E 14 and half of 84 p.


Use doubling and halving as a strategy in dividing by $2,4,8,5$ and 20 , e.g. $115 \div 5$ as double $115(230) \div 10$.

## Grouplng

Divide numbers by $10,100,1000$ to obtain decimal answers with up to three places, e.g. $340 \div 100=3.4$.
Use the 10th, 20th, 30 th ... multiple of the divisor to divide friendly 2 -digit and 3 -digit numbers by single-digit numbers.
e.g. $186 \div 6$ as $30 \times 6(180)$ and $1 \times 6(6)$.

Find unit and non-unit fractions of large amounts, e.g. $3 / 5$ of 265 is $3 \times(265 \div 5)$.
Use the bar model when representing fractions of numbers.

| 265 |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| 53 | 53 | 53 | 53 | 53 |

Using number facts
Use division facts from the times tables up to $12 \times 12$ to divide mutiples of powers of ten of the divisor, e.g. $3600 \div 9$ using $36 \div 9$.

## Year 5

## Written Multiplication

## Written Division

Short multiplication of 2-digit, 3-digit and 4-digit numbers by 1-digit numbers.


Children may use column multiplication to solve money problems though encourage children to use jottings and mental methods. Be clear on the place value of pounds and pence.

Multiply a 2-digit number by a 2-digit number, progressing to the multiplication of 2 -digit number by a 3 -digit and 4 -digit number.


Ensure that children have a clear understanding of why the zeroplace holder - is needed. Children can use grid multiplication to check their solutions; being confident in both methods is great.

## Multiplying fractions

Children should connect multiplying proper and mixed fractions by a whole number to repeated addition and use visuals to aid understanding.

Visual images are essential to multiplying and dividing fractions.
The closer division is linked to multiplication the better.

Short divison of 3-digit and 4-digit numbers by single-digit numbers.

$$
6 \begin{array}{|c}
1264 \\
\hline{ }^{1} 58^{3} 4
\end{array}
$$

Children who are struggling to grasp the method from year 4 may use place value counters to aid understanding.

$163 \div 3$ is seeing "how many groups of 3 " are in 163 . Children can use counters to make groups of 3 . $163 \div 3=121$.
Children can use counters to help understand exchanging where a full group cannot be made. $345+3=$

$432 \div 5=86$ r2

$$
\frac{1}{6} \times 4=\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}=\frac{4}{6}=\frac{2}{3}
$$

$5 \longdiv { 4 6 r 2 }$
Children will learn to express the remainder as a fraction - the remainder divided by the divisor.

$$
432 \div 5=86^{\frac{2}{5}}
$$

## Year 6

## Mental Multiplication

## Doubling and halving

Double decimal numbers with up to 2-places using partitioning, e.g. 36.73 doubled is double 36 (72) plus double 0.73 (1.46).


Use doubling and halving as strategies in mental multiplication.


## Doubling and halving

Halve decimal numbers with up to 2 -places using partitioning. e.g. half of 36.86 if half of $36(18)$ plus half of $0.86(0.43)$.


Use doubling and halving as strategies in mental division, e.g. $216 \div 4$ is half of 216 (108) and half of 108 (54).

## Grouping

Use 10th, 20th, 30th, ... or 100th, 200th, 300th ... multiples of the divisor to divide large numbers, e.g. $378 \div 9$ as $40 \times 9=360$ and $2 \times 9=18$
so, the answer is 42.

$$
\begin{aligned}
& 378 \div 9=\quad \times 9=378=9 \times \ldots=378 \\
& 9 \times 40=360 \\
& 9 \times 2=18 \\
& 9 \times 42=378 \text { so } 378 \div 9=42
\end{aligned}
$$

Division as grouping. i.e. the inverse of multiplication, is a key concept.

## Mental Division

## Grouplng

Use partioning as a strategy in mental multiplication, as appropriate, e.g. $3060 \times 4$ as $(3000 \times 4)+(60 \times 4)$ or $8.4 \times 8$ as $8 \times 8(64)$ and $0.4 \times 8(3.2)$
Use factors in mental multiplication,
e.g. $421 \times 6$ as $421 \times 3$ (1263) doubled (2526)
or $3.42 \times 5$ as half of $3.42 \times 10$.
Mutliply decimal numbers using near multiples by rounding.
e.g. $4.3 \times 19$ as $4.3 \times 20(86-4.3)$.

## Using number facts

Use times tables facts up to $12 \times 12$ in mental multiplication of large numbers or numbers with up to two decimal places.
e.g. $6 \times 4=24$ and $0.06 \times 4=0.24$.

## Using number facts

Use division facts from the multiplication tables to help divide decimal numbers by single digit numbers
e.g. $9.6 \div 8=96 \div 8 \div 10=12 \div 10=1.2$

## Written Multiplication

## Written Division

Short multiplication of 2-digit,
3 -digit and 4 -digit numbers by 1 -digit numbers.

Long multiplication of 2-digit, 3250 $\begin{array}{r}\times \quad 26 \\ \hline\end{array}$ 19500
$+\quad 65000$
84500
3 -digit and 4-digit numbers by 2 -digit numbers.


Short versions of multiplication and division are more important and useful than the long versions.

Short division of 3-digit and 4-digit numbers by single-digit numbers.

6 | 1264 |
| ---: |
| $715 \frac{3}{4} 4$ |

Short multiplication of decimal numbers using $\times 100$ and $\div 100$, e.g. $13.72 \times 6$ as $1372 \times 6 \div 100$.

Short multiplication of money,

$$
\begin{array}{r}
£ 1.27 \\
\times \quad 6 \\
£ \frac{7.62}{14}
\end{array}
$$

e.g. $£ 13.72 \times 6$ or $£ 23.67 \times 3$.

Multiply proper and improper fractions by fractions, by using images to support understanding. Children should be encouraged to see the relationship between $x$ and of.


$$
\begin{aligned}
& \frac{1}{3} \times \frac{1}{2}=\frac{1}{6} \\
& \frac{1}{3} \text { of } \frac{1}{2}=\frac{1}{6}
\end{aligned}
$$

$$
\frac{4}{5} \times \frac{2}{3}=\frac{8}{15}
$$

When children understand, they can apply to larger non-unit fractions.

Use knowledge of factor pairs to solve division calculations including 3-digit and 4-digit by a 2-digit number then use short division

$$
7,848 \div 24
$$

$$
\begin{aligned}
& 7848 \div 4 \div 6 \\
& 7848 \div 2 \div 2 \div 6 \\
& 2848 \div 2 \div 12
\end{aligned}
$$

Long division of 3-digit and 4-digit numbers by two-digit numbers.

| 200+50+1 | $\times 10 \times 10$ |  |  | Children write out the multiples and |
| :---: | :---: | :---: | :---: | :---: |
| $1 5 \longdiv { 3 7 6 5 }$ | 15 | 0 |  |  |
| 300 | 30 | 0 | 0 | add a column for $\times 10$ and $\times 100$. |
| 3000 | 45 | 0 |  | They pick the closest number to that |
| 765 | 60 | 0 | 0 | being divided, writing the number of |
| 750 | 75 | 0 | 0 | groups along the top and subtracting |

Divide fractions by whole numbers by using $\frac{1}{6} \div 3$ imagery and context to support understanding.

$$
\frac{1}{6} \div 3=\frac{1}{6} \div \frac{3}{1}=\frac{1}{6} \times \frac{1}{3}=\frac{1}{18}
$$



